AN EXPERIMENT TO MEASURE SURFACE CURRENTS PRESENT IN PLASMON EXCITATION FIELDS

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Western Carolina University in partial fulfillment of the
requirements for the degree of Master of Science in Technology.

By

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To my mom and dad

who always believe in me

despite common sense

ACKNOWLEDGEMENTS

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ABSTRACT

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This thesis seeks to help prove an existing theory in the field of surface plasmonics. In their paper Surface Plasmon Excitation and Non-Zero Induced Surface Current Density, Dr. Weiguo Yang of Western Carolina University and Dr. Michael Fiddy of the University of North Carolina at Charlotte showed that, contrary to conventional understanding, there exists a non-zero electric current on the surface of a metal (or other media) while a plasmon charge-inducing electromagnetic field is present. This current is also not present when the field is removed, even though the plasmon wave is self-sustained. Following the publication of the work of Dr. Yang and Dr. Fiddy on the presence of these surface current densities in plasmon fields, an experiment will be conducted to detect and measure this current. These surface current densities have not yet been measured, as their existence has only been shown mathematically. If the current can be shown to exist, Dr. Yang and Dr. Fiddy's theory will be validated and will open the door to new areas of research in the existing field of Surface Plasmonics and in the general study of electromagnetic waves. This project will consist of the construction of an experiment setup to measure an induced surface current in the presence of a plasmon excitation field. The current-detecting apparatus will also be explored as a new type of electronic sensor.

CHAPTER 1: INTRODUCTION

1.1 Overview

The study of surface plasmons may not be among the most well-known, but recent advances in optics and electromagnetics may open this field to much broader study. A plasmon wave exists when an electromagentic wave (such as a laser or light from the sun) strikes a conductive surface at a specific angle. The plasmon wave represents a loss of power from the electromagnetic wave that is reflected. Therefore, if light reflected off a surface shows a significant power loss, then the angle of the light striking the surface is at a specific, known value. Alternatively, if the reflected power from a light shows a power loss, then the surface is known to be at a specific angle relative to a fixed source of light.

1.2 Objective, Significance, and Limitations

The objective will be to develop and test a method to measure changes in electric current on the surface of a gold film as the film has a plasmon wave excited upon it. The electric current in this set of conditions has previously not been considered to exist. No publication has made note of such a current being created. Recently, the current has been shown to exist mathematically, albeit very small [1]. A sensor could utilize this phenomenon to detect surface plasmonic resonance without the need to measure reflected power.

The power of the plasmon wave is proportional to the power of the electromagnetic wave propagating the plasmon, and the output of the laser available for testing only has an output of 1.5 milliWatts. If the surface current can be shown to exist under plasmon resonance conditions in a low-power environment, it follows that a surface plasmon resonance sensor could be developed that requires very little input power.

1.3 Outline

The thesis will be divided into several chapters. Chapter two describes previous work done in the field of plasmonics as well as current practical applications of plasmon excitations. This chapter will also discuss the implications of a surface current being present in a plasmon excitation. Chapter three presents the methodology through which the experiments are designed and carried out. Chapter four contains the results of the experiments and a discussion of the prevalent data. Chapter five will present a conclusion from the data as well as discuss future work that can be done on this topic.

CHAPTER 2: LITERATURE REVIEW

Experiments from as early as 1975 seek to show that the dispersion of surface plasmons can be performed in a laboratory. The dispersion relation is derived through relating the vector of a surface wave that is propagated through a metal to its angular frequency [2]. A process is designed, in a more contemporary text, to excite a surface plasmon on a metal film, and its relation to the incident radiation is described. The experiment performed shows the change in reflectivity of a silver film, but mainly serves as a demonstration of a technique to observe a surface plasmon in a metal film. Pitarke et al extensively describe the theory of surface plasmons and surface-plasmon polaritrons [3]. The physics of plasmon excitation conditions and the understanding of the nonexistence of surface currents are well-understood. An exploration of these different environments to increase the current level on the metal surface could be explored [4]. This experiment only explores one method of creating a surface plasmon wave.

Some other experiments and lectures describe some techniques will be utilized in this thesis. A 2008 report describes conventional techniques of measuring electron mobility on the surface of ultra-thin evaporated gold films. The experiment described uses an aucousto-electric method to measure the electron mobility and diffusion on the surface of a film. This is used in our experiment to calculate the expected magnitude of the surface current [5]. The current density is expected to exist at such a small magnitude that the electron mobility of the film needs to be taken into consideration. The mobility links the electric field to the average velocity of the material carrier. Once the current density is calculated, the conductivity and resistivity of the semiconductor (conductor, in our experiment) can be calculated [6].

Surface plasmonic waves are a charge density wave the propagates along the interface of two different media. These waves are self-sustained and do not require an input electromagnetic field. However, when an electromagnetic field is applied, it is understood that these surface plasmon waves make up for the normal component of an applied electric field being discontinuous, but the waves do not carry a current on the surface. This is evidenced by the fact that the magnetic component of the applied field is continuous. Dr. Yang and Dr. Fiddy's calculations show that while a surface current does not exist without an applied electromagnetic field, a non-zero current can and does coexist with the surface plasmon wave when the field is applied. In addition, the calculations show that this surface current exists in general cases where P- and S-type polarizations are applied and in non-resonant excitations [1]. As shown in Yang and Fiddy's previous publication, a non-zero surface current accompanies an excited surface plasmon wave when an electric field is applied. The surface plasmon wave persists when the electric field is removed, but the accompanying current does not. Further examination shows the depth of the "skin effect" of a surface plasmon polarization in metal conductors and the surface source distributions as the result of SPP excitations under general excitation conditions [7]. These two papers describe the phenomenon that this thesis will explore.

As an example of contemporary applications, Biacore Life Sciences describes a system they have developed commercially to monitor biomolecular interactions using surface plasmon resonances. The process uses a typical plasmonic setup but is dependent on the refractive index of different proteins to create the plasmon wave (e.g. a specific protein would refract the light sufficiently enough to create (or not create) the plasmon wave) [8]. This application could be improved if a current were detected directly on the testing surface, instead of having to measure the reflected power of an applied electromagnetic wave. In another example, Dr. Yang explores of the applications of using a non-zero surface current in a surface plasmon wave to directly detect electromagnetic fields. This presentation was made by Dr. Yang for a previous project, funded by the National Science Foundation, for the Center for Metamaterials. This project proposal explores the next step after proving that a non-zero surface current exists. Some applications described include optical sensing and light detection, especially in wavelength ranges where direct detectors are expensive or difficult to use [9].

In conclusion, a surface current exists mathematically during surface plasmon resonance (SPR). If it can be detected and measured, it can be used in applications that utilize the plasmonic phenomenon. Specifically, the surface current could be used to directly indicate a plasmon excitation on a surface, instead of the contemporary practice of detecting this excitation indirectly, through the loss in reflected power of the stimulating electromagnetic wave.

CHAPTER 3: DESIGN METHODOLOGY

3.1 Test Setup

3.1.1 Explanation

Under the current understanding of the behavior of a plasmon wave, its existence suggests only a loss of power in the reflected electromagnetic wave that can be detected. If the plasmon wave also carried a non-zero surface current, its existence could be detected directly on the surface of the material an electromagnetic wave is incident to, instead of having to measure the reflection of the electromagnetic wave. This experiment seeks to measure a surface current during a plasmon excitation by detecting a change in voltage between two points. Under normal circumstances, the electric potential at two close points on a gold film should be the same. Surface plasmon resonance is theorized to make a change in electron mobility. The electron mobility, μ, in a given material can be derived from the equation for a material's electron drift velocity, $u = \mu/E$, where E is the applied electric field density. The drift velocity is also defined as

$$u = \frac{I}{nAq}$$

where I is the current through a material, n is the number of free electrons per cubic meter, A is the cross-sectional area of the material, and q is the charge of the electron, $-1.6 \times 10^{-19} C$. Gold has a density of 19.3 $\frac{g}{cm}$, an atomic weight of $3.270 \times 10^{-25} kg$, and one free electron per atom. Therefore, n equals 2.28×10^{27} electrons per cubic meter.

The cross-sectional area for the experimental film is simply its width, 2 cm times its thickness, 10 μm, resulting in $2 \times 10^{-7} m^2$. If we can assume the current I to be $1A$, then the drift velocity is

$$u = \frac{1A}{(2.28 \times 10^{27} m^{-3})(2 \times 10^{-7} m^2)(1.602 \times 10^{-19} C)}$$
$$u = \frac{1A}{(4.56 \times 10^{20} m^{-1})(1.602 \times 10^{-19} A \times s)}$$
$$u = \frac{1}{73.05 \frac{s}{m}}$$
$$u = 1.37 \times 10^{-2} \ \frac{m}{s}$$

We can use the assumed current $1A$ here, or we can simply leave current a variable to get the equations

$$\mu = 1.37 \times 10^{-2} E \qquad or \qquad \mu = \frac{IE}{73.05}$$

If there were a current between two points on the gold film during surface plasmon resonance, a small change in voltage between the two points should be detected.

3.1.2 Equipment

- Research Electro-Optics 30025 635nm (red) HeNe 1.5mW laser system
- TPI 192 II Digital Multimeter
- Stanford Research Systems SR830 DSP Lock-in Amplifier
- Stanford Research Systems SR540 Chopper
- Anritsu MA9302A Optical Power Sensor
- Anritsu ML910B Optical Power Meter
- Newport FK-18690 Half-Waveplate Package
- ThorLabs LPVISE100-A Linear Polarizer
- ThorLabs PS852 Equilateral Prism
- Cascade Microtech DCP 115-R Surface Probe
- Cargille index-matching (n=1.6097) immersion liquid

3.2 Timeline

- April 2017:

  Submit thesis proposal and continue researching background information about plasmon experiments

- May 2017:

  Design experimental setup

- June-August 2017:

  Perform surface current-measuring experiments

  Make adjustments to experimental design and continue experimentation

- September-December 2017:

  Analyze collected data

- January-March 2018:

  Write thesis and prepare presentation(s)

- April 2018:

  Defend thesis and submit for publication

3.3 Experimental Procedure

This experiment resembles a typical optics setup. A laboratory laser serves as the electromagnetic wave source. Its light passes through a polarizer, followed by a waveplate. The light is then refracted though a prism, reflected off a gold film back through the prism, and into an optical power sensor. Figure 3.1 provides a diagram of the laser beam passing through the prism. The film has been prepared by evaporating gold onto a glass plate. This plate is then adhered to the prism with a refraction index-matching oil. The gold is actually on the outside of the plate (relative to the prism) so the laser light is reflected off the back. With this in mind, changes to the angle of the laser incident to the prism must take into account the refractory index of the glass. For the sake of record-keeping, the angle of the light incident to the film is recorded only as the angle of the light incident to the prism. The internal angle of the light is what excites the plasmon wave and is calculated using

$$\theta_{internal} = sin^{-1}\left(\frac{sin(\theta_{external} - A)}{n}\right) + A \tag{3.1}$$

where $n = 1.62$, the known refractive index of the glass, and $A = 60°$. The angle at which surface plasmon resonance occurs is known as the critical angle, and is expressed relative to the normal angle.

Before finding the angle for the plasmon excitation, the laser needs to be aligned. The light is first polarized to ensure that the light is a single directional wave when it reflects off of the film. This is done by adjusting the polarizing waveplate until the optical power reflected is minimized. Once the angle of polarization is set, a second waveplate is rotated until the reflected power is at a maximum. This insures that the wave is polarized and horizontal when it is reflected off of the plate. These angles are not adjusted again. The table setup is illustrated in Figure 3.2. In the first experiment, the critical angle needs to be established. This done by simply adjusting the plate in 0.5° increments. Zero (or 360) degrees represents the normal angle of the plate (i.e.the plate reflecting light directly back

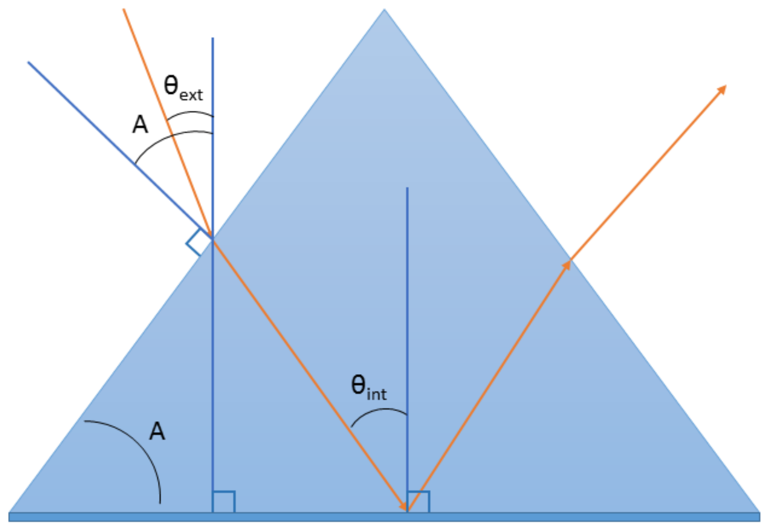

Figure 3.1: Illustration of the refractive difference between internal and external angles

at the source). The angle at which the reflected power is at a minimum is the angle where plasmon excitation occurs. The angle is adjusted from 311° to 348° because the input angle becomes too steep to strike the plate before this range, and after the range the reflected light is too close to the laser setup for the optical power sensor to measure. The reflected optical power is recorded at each increment and the trial is repeated five times. Plots of the measured reflected power as a function of external angle for each trial are shown in Figure 3.3. Notice that there is very consistent agreement between trials. The alignment process shows that there is a minimum most often at an external angle of 329°, or −31° from normal.

For the next experiment, the range of angles measured spans two degrees on either side of the angle at which the plasmon excitation was established. The probes pictured in Figure 3.8 are in contact with the plate while the plate is moved through the full range of reflection. This is illustrated in Figure 3.4.

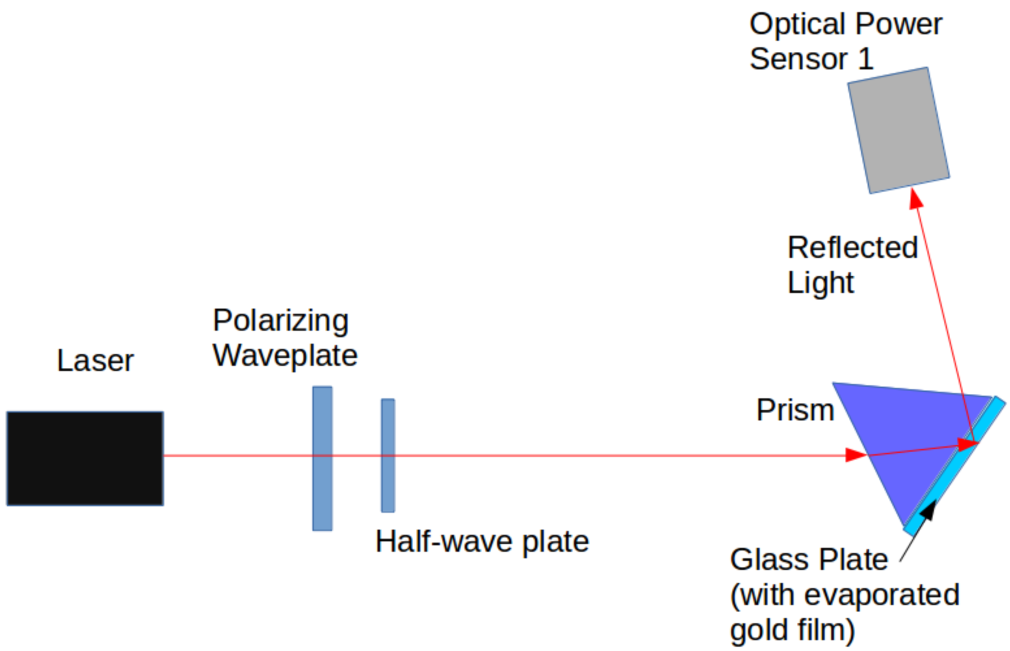

Figure 3.2: Alignment Setup

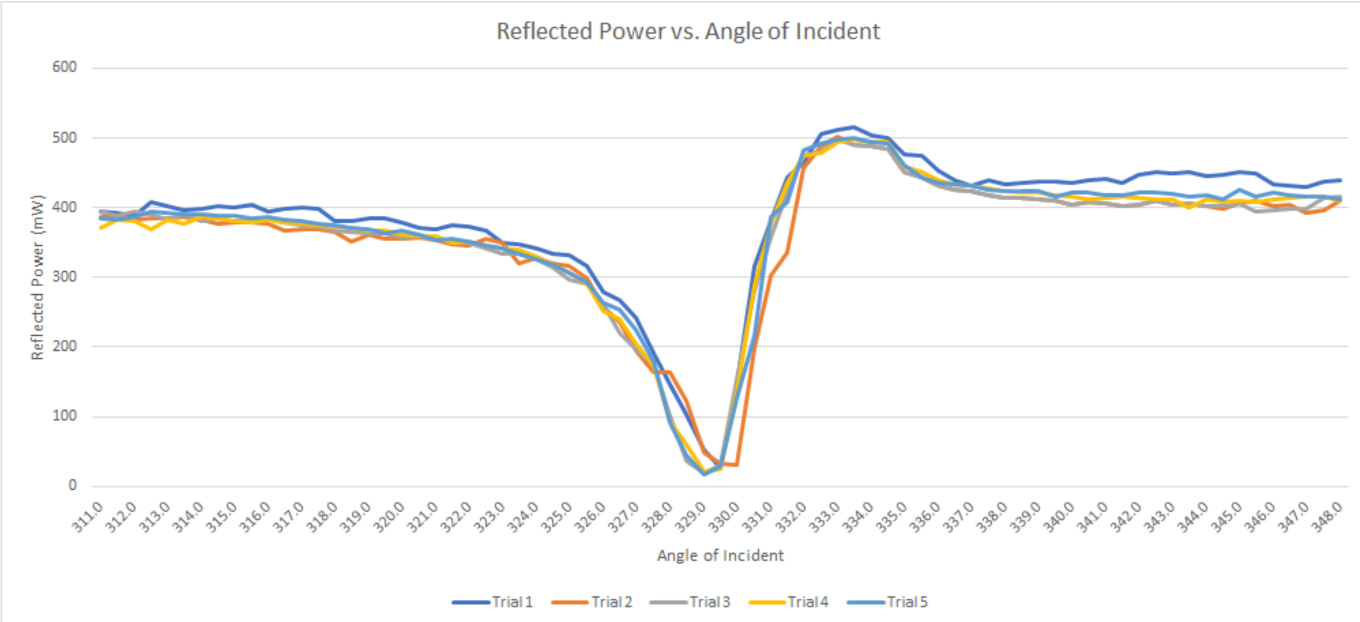

Figure 3.3: Angle (degrees) vs. Reflected Power (microWatts)

The standard ”square” gold film plate was replaced with a gold custom-cut pattern on a plate. Pictured in Figure 3.5, this pattern was designed to have a very small width to ensure that currents on the surface follow a set path. In addition, the custom cut plate has its own probing attachment, so that the pin type probes do not have to be adjusted with each change in angle. This probing attachment is connected to a multimeter and the difference in voltage is recorded.

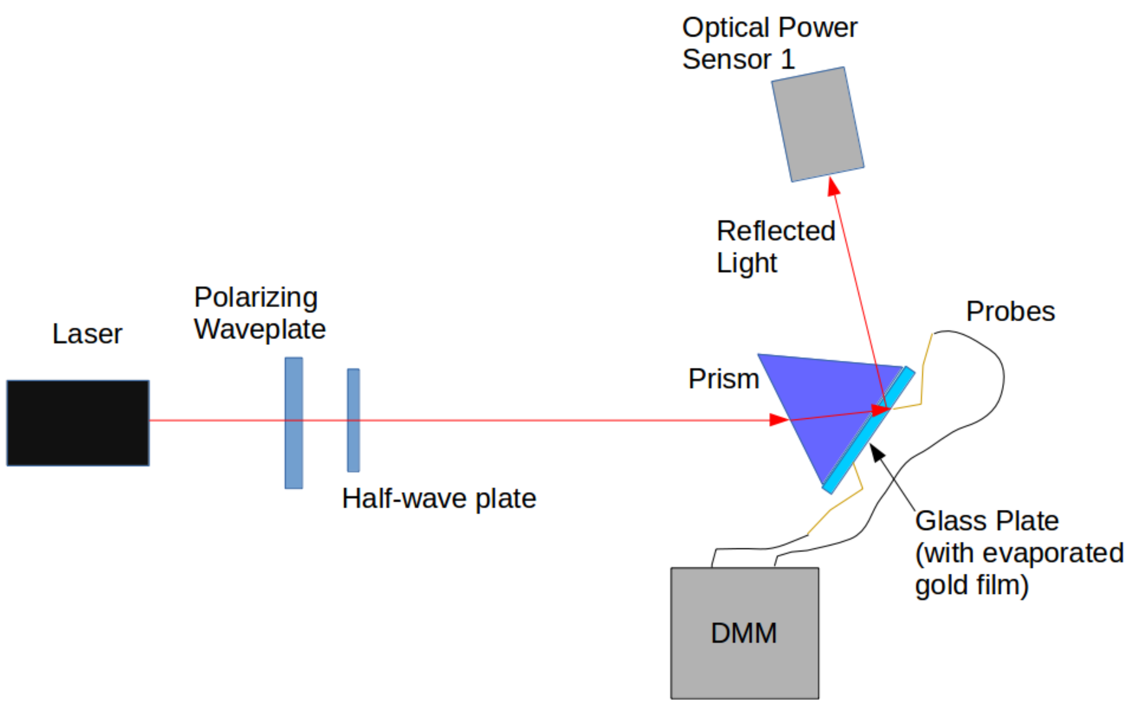

Figure 3.4: Experiment 2

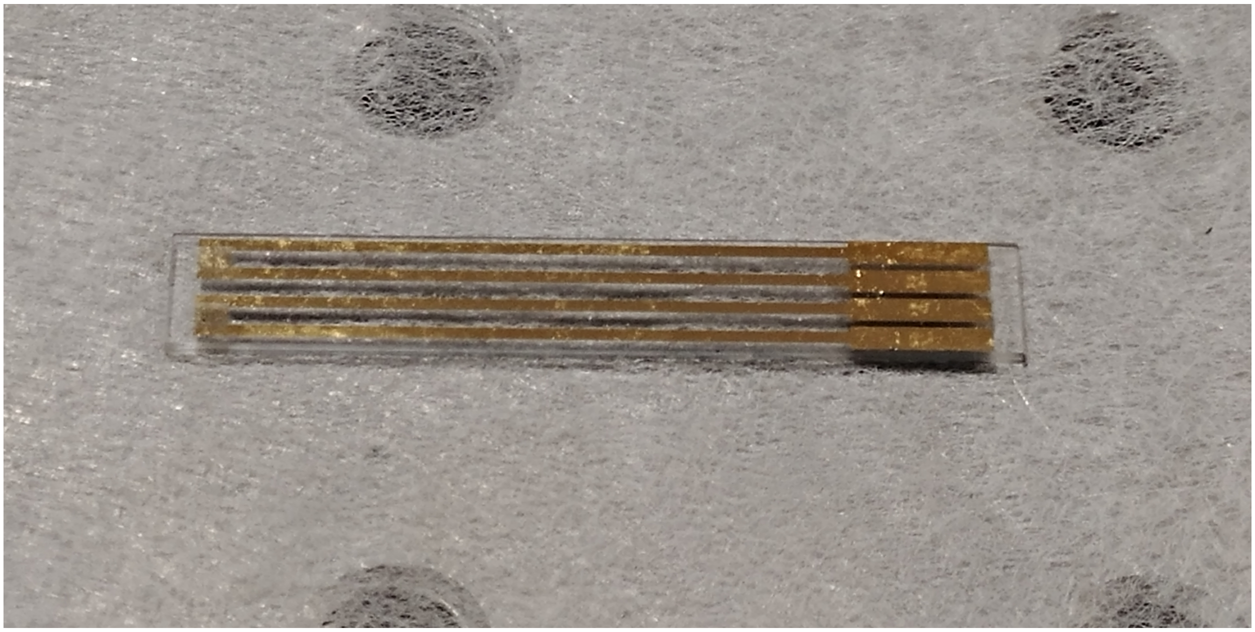

Figure 3.5: Custom Cut Plate

For a final test of the differences in voltage, the probes measuring the difference in voltage are connected to a lock-in amplifier (the large device pictured in Figure 3.7). This amplifier is synchronized with a wheel that ”chops” the laser at a specific frequency. This process is done so that the voltage difference is read for fractions of a second at time, thus minimizing the noise that occurs when a voltage is read continuously for a long time. The amplifier outputs the difference in voltage as a magnitude and its phase shift. This experiment was done with the angle of reflection fixed on the plasmon excitation. Several readings were taken with the laser off, with the standard setup with horizontal polarization, and with the half-wave plate adjusted 45°. (Note that a 45° change on a half-wave plate rotates that plane of polarization 90°.) The probes on the etched pattern are pictured in Figure 3.8.

In a different type of experiment, the transmitted power was recorded instead of the voltage on the surface of the film. Transmitted power is from the light that passes through the gold film instead of being reflected. Light passes through the film up until the internal angle of the prism becomes too steep. At this point, nearly all light is reflected by the film.

This experiment was performed twice, once with a horizontal polarization and a second time with a vertical polarization. This setup is illustrated in Figure 3.6.

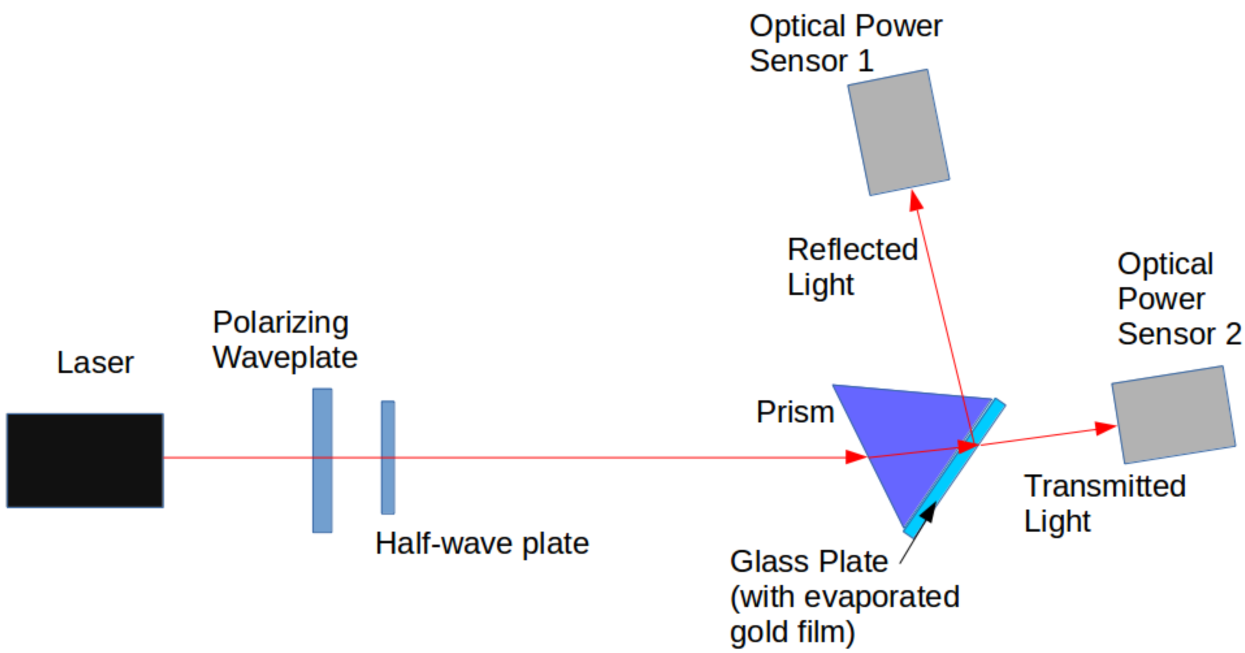

Figure 3.6: Experiment 3

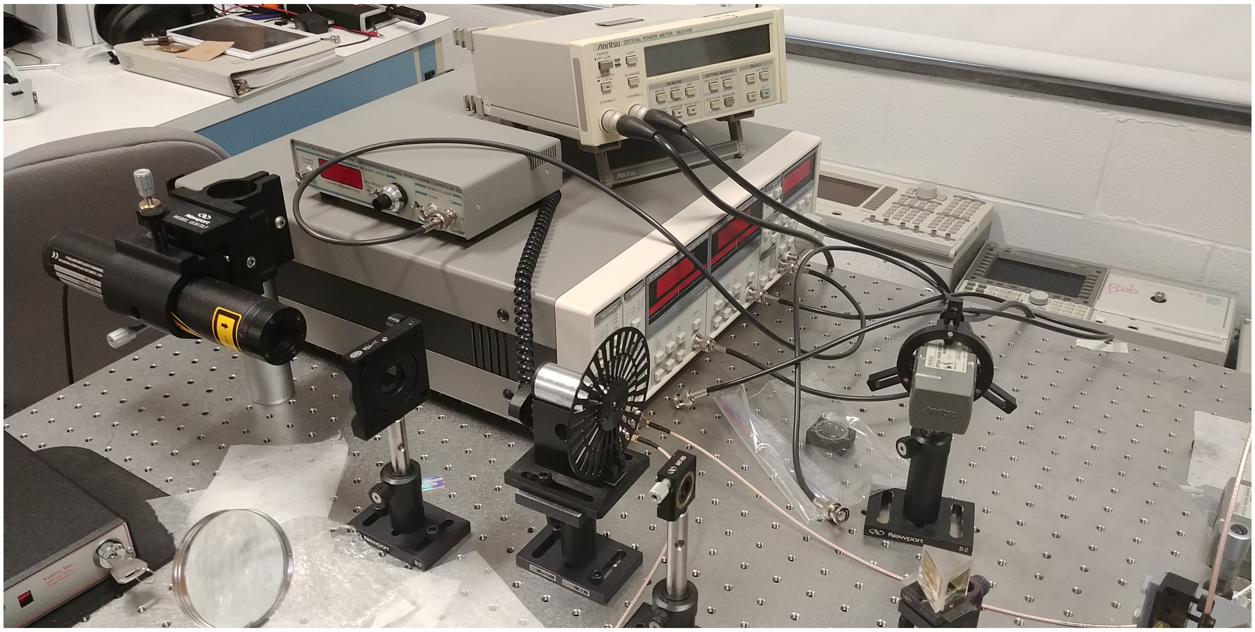

Figure 3.7: The test setup with Power meter and Lock-in amplifier

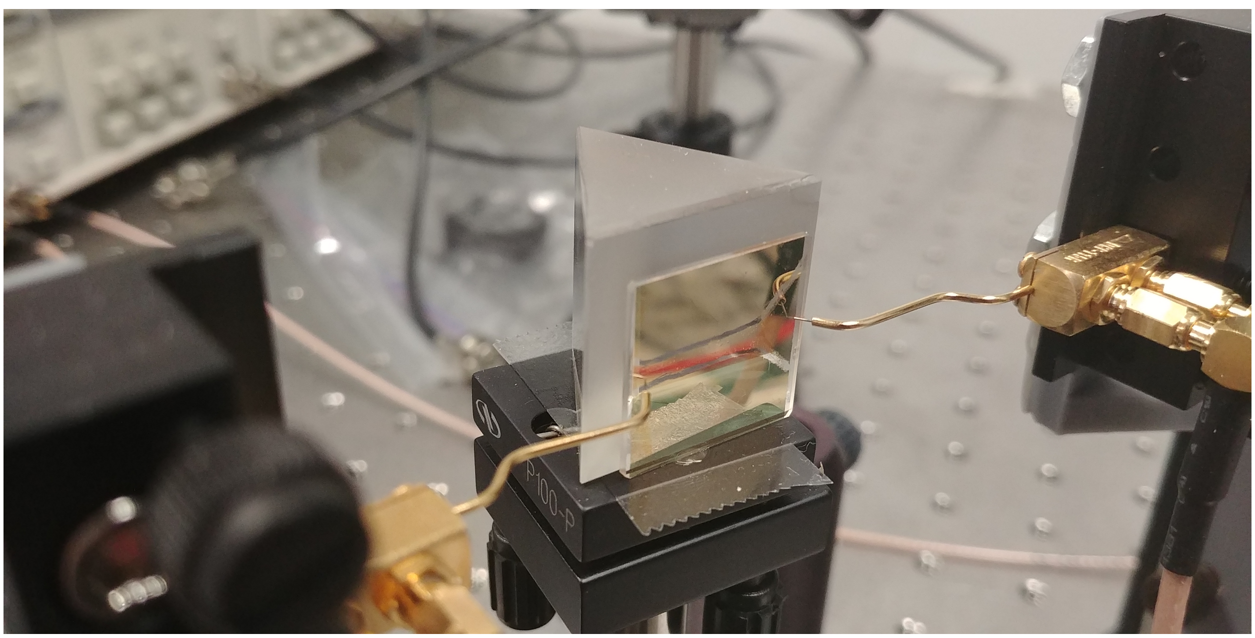

Figure 3.8: The probes in contact with the gold film

CHAPTER 4: RESULTS

During alignment, it was shown that the plasmon excitation is expected to occur at about 329.5°. This number is used in the subsequent experiments to ensure the appropriate angle is used. It should be noted that the reflected power level is expected to be at around 20 μW or less. Since the changes in the power level are significant between half-degrees, a power level less than 20 μW is a better indicator of SPR than the nominal angle; especially since the mounted prism is prone to shifting between experiment setups.

The results from the first experiment showed the the digital multimeter (DMM) did not have the resolution to record changes in voltage. Once the DMM was switched out for the lock-in amplifier it was certain that the results recorded were more precise, but inconclusive nonetheless.

In the second experiment, it was shown that the differences in voltage between the two probes on the surface of the plate are not indicative of change during a plasmon excitation. There is roughly the same difference in potential outside the plasmon excitation region as there is in it. Note in Figure 4.1 that in each trial, there is very little change in differences of potential regardless of the angle of incidence. Plasmon Resonance is expected at 0°and is extinct at $-2°$ and 2°.

The third experiment, measuring transmitted power, showed that transmitted power falls off to zero before the angle of the light reaches plasmonic resonance. The results in Figure 4.2 demonstrate that transmitted power is not an indicator of changes in electric potential on the surface of the material. Note that even though surface plasmon resonance occurs at 329°, indicated by the drastic loss of reflected power in the horizontal polarization, the transmitted power falls off to zero before this region occurs. Vertical polarization does not see a drastic drop in reflected power at the expected critical angle, as this orientation of polarization is the "worst case" for SPR.

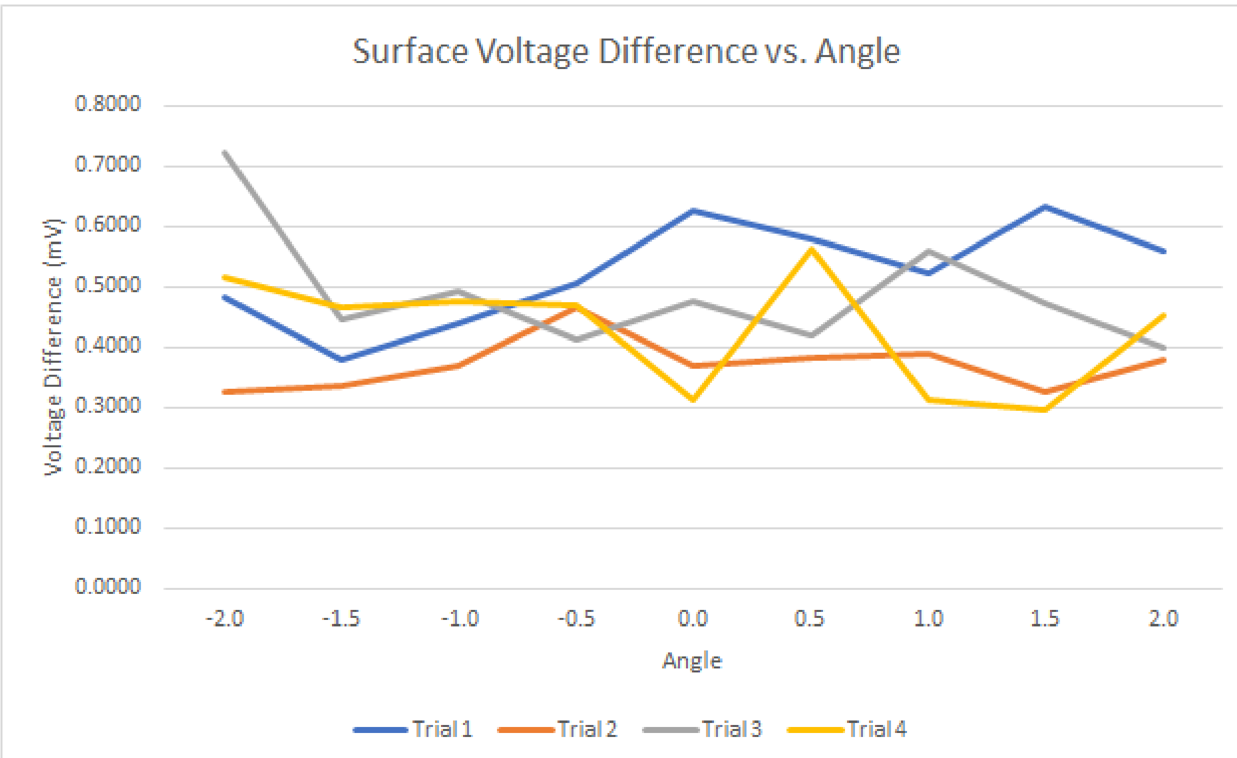

Figure 4.1: Experiment 2: Angle vs. Voltage difference

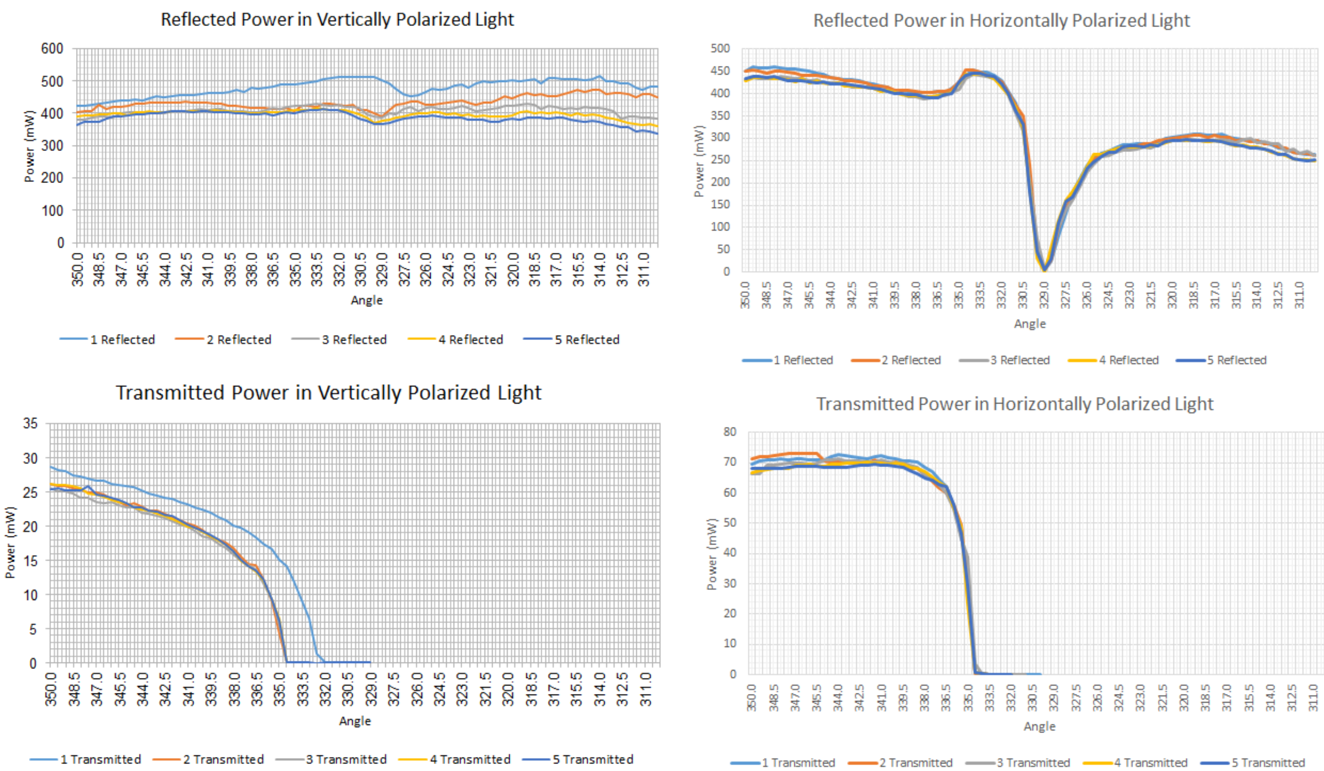

Figure 4.2: Experiment 3: Reflected and Transmitted Power in Horizontal and Vertical Polarization

Unfortunately, these results to not strongly indicate that a current exists on the surface of the film during a plasmon excitation. Furthermore, the results from the third experiment are not encouraging. The magnitude of the difference in potential is about the same regardless of whether the laser is on or off. The complete results of the all experiments are included in the appendix.

CHAPTER 5: CONCLUSION AND FUTURE WORK

In conclusion, these experiments are not a strong indicator that a surface current exists in a surface plasmon excitation. This experiment utilized the most sensitive equipment available but failed to detect a voltage difference indicative of a stimulated current. This same experiment may be performed with even more sensitive equipment to attempt to find a positive result. If this experiment were performed with a more powerful laser, or possibly with a laser with a shorter wavelength, the plasmon field would have greater power and thus may have a current more easily detected.

Another source of error may be due to the skin effect. If the current densities are very small, the resistance of the gold may inhibit them from being detected in this experiment. This was not taken into account because the gold film is only a few Angstroms thick, but it can be thinner. The mathematics supporting the existence of the electric current densities assume the surface to not be a factor.

Future experiments could also explore other methods to detect changes in surface current. This experiment only tested for changes in differences of electric potential.

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Appendices

APPENDIX A: COLLECTED DATA

Table A.1 shows data gathered from alignment. On the left is the angle of the light incident to the gold film and the five trials show the reflected power in milliwatts.

Table A.1: Optical Alignment

Angle	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
311.0	394	387	394	371	385
311.5	392	388	388	383	382
312.0	388	383	394	380	386
312.5	408	384	391	370	395
313.0	402	385	383	383	392
313.5	397	387	390	377	391
314.0	399	382	381	386	390
314.5	403	377	384	385	388
315.0	401	379	380	381	389
315.5	404	378	381	379	385
316.0	395	376	384	382	387
316.5	398	367	379	379	382
317.0	400	370	375	376	381
317.5	399	369	373	375	377
318.0	381	366	368	374	375
318.5	380	352	365	371	372
319.0	385	362	363	367	369
319.5	384	356	363	368	363
320.0	378	355	358	361	367
320.5	371	357	360	359	361
321.0	370	354	355	360	354
321.5	375	348	351	349	355
322.0	374	345	349	352	352
322.5	368	356	342	346	346
323.0	350	349	333	342	341
323.5	348	321	334	339	334
324.0	342	329	326	330	327
324.5	333	321	314	318	318
325.0	331	316	296	307	306
325.5	316	299	290	290	293
326.0	280	258	262	252	264
326.5	267	237	220	241	254
327.0	242	196	198	204	224
327.5	192	164	173	173	180

Table A.1 continued from previous page

Angle	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
328.0	147	164	101.4	93.3	91.1
328.5	103	123	37.5	60.1	44.6
329.0	52.2	48.7	19.8	20.9	16.7
329.5	26.5	31.8	30.7	25.1	28.1
330.0	146	30.9	154	135	127
330.5	317	199	282	289	217
331.0	380	303	358	377	386
331.5	443	336	421	436	409
332.0	465	458	475	475	483
332.5	506	489	480	479	493
333.0	511	502	499	495	498
333.5	515	490	490	499	500
334.0	505	489	489	494	495
334.5	500	485	485	494	492
335.0	476	452	452	460	462
335.5	475	443	443	451	444
336.0	454	431	431	440	436
336.5	440	425	425	434	433
337.0	432	424	424	431	431
337.5	439	418	418	427	426
338.0	433	414	414	423	423
338.5	435	415	415	421	424
339.0	437	412	412	421	423
339.5	438	411	411	419	417
340.0	435	405	405	416	421
340.5	440	408	408	413	421
341.0	441	407	407	414	419
341.5	436	402	402	417	419
342.0	447	404	404	415	422
342.5	451	411	411	413	421
343.0	450	404	404	412	420
343.5	451	406	406	400	417
344.0	446	403	403	412	418
344.5	447	399	402	408	413
345.0	452	407	406	410	425
345.5	450	410	395	409	416
346.0	433	403	396	412	422
346.5	432	404	399	414	418
347.0	429	393	398	417	417
347.5	437	396	414	416	417
348.0	440	410	416	411	413

Table A.2: Experiment 1

Laser Off		Horizontal Polarization		45° Polarization	
R	θ	R	θ	R	θ
0.041	114.20	0.087	79.92	0.021	98.88
0.025	56.52	0.024	82.77	0.047	35.81
0.014	77.90	0.020	84.12	0.081	67.63
0.023	84.31	0.034	-161.44	0.048	164.42
0.062	70.48	0.085	173.71	0.078	49.16
0.038	51.52	0.071	63.88	0.024	78.52
0.044	61.38	0.026	-133.11	0.022	96.09
0.026	75.78	0.040	80.95	0.026	92.18
0.044	95.34	0.052	65.59	0.020	90.94
0.063	85.78	0.051	130.19	0.020	103.34
0.062	88.41	0.025	99.39	0.049	119.70
0.025	54.02	0.056	98.40	0.055	71.34
0.021	88.17	0.050	94.15	0.033	70.68
0.037	78.11	0.064	51.80	0.061	84.97
0.077	45.59	0.009	61.92	0.034	93.42

Table A.2 shows the experiment wherein the angle of incidence was fixed at SPR and the laser was turned off, polarized horizontally, and polarized vertically. The R (from the lock-in amplifier) is the amplitude of the difference in voltage and the θ is the phase difference.

Table A.3: Experiment 2

Angle	Trial 1	Trial 2	Trial 3	Trial 4
-2.0	0.4827	0.3252	0.7246	0.5156
-1.5	0.3789	0.3365	0.4460	0.4653
-1.0	0.4410	0.3692	0.4922	0.4751
-0.5	0.5051	0.4663	0.4137	0.4700
0.0	0.6250	0.3707	0.4753	0.3127
0.5	0.5800	0.3829	0.4191	0.5636
1.0	0.5246	0.3906	0.5611	0.3112
1.5	0.6343	0.3273	0.4721	0.2963
2.0	0.5597	0.3804	0.3990	0.4535

Table A.3 shows the four attempts to "sweep" the 4 degrees across the SPR region. The values at each angle are listed in millivolts.

Tables A.4 and A.5 show the five attempts to sweep the full range of reflection and gather reflection and transmission power in mW with horizontal polarization. Note that transmission values less than $5\mu W$ are omitted.

Table A.4: Experiment 3: Horizontal Polarization

	Trial 1		Trial 2		Trial 3	
Angle	Reflected	Transmitted	Reflected	Transmitted	Reflected	Transmitted
350.0	451	69.5	451	71.4	432	66.5
349.5	460	70.7	452	71.9	436	66.5
349.0	459	71.1	450	72.1	435	69.1
348.5	459	71	446	72.5	435	69.2
348.0	460	71.3	450	72.7	434	69.7
347.5	457	71.1	450	73	438	70.1
347.0	456	71.4	448	73.2	437	70.1
346.5	456	71.2	446	73.1	434	70
346.0	452	71.1	441	73.2	434	69.7
345.5	450	71.1	442	73.2	430	69.9
345.0	447	70.9	442	70.5	432	71
344.5	443	72.2	439	70.4	425	70.9
344.0	436	72.6	436	70.3	425	71.3
343.5	435	72.4	433	70.3	425	70.8
343.0	432	72.1	429	70.2	421	70.8
342.5	432	71.8	428	70.2	416	70.7
342.0	428	71.5	426	70.1	417	70.4
341.5	425	72.2	425	70.9	415	70.2
341.0	423	72.4	417	70.4	412	71.1
340.5	417	71.8	417	70	408	70.4
340.0	416	71.2	414	70.2	407	70
339.5	405	70.5	408	69.1	404	70
339.0	407	70.8	407	68.2	401	69.2
338.5	405	70.2	408	68	393	68.4
338.0	403	68.4	405	65.9	394	66
337.5	402	67	402	63.9	389	64.9
337.0	404	64.5	402	61.7	391	62.8
336.5	406	62.1	406	60.2	396	60.2
336.0	401	55.8	406	56	396	54.6
335.5	413	45.4	411	49.4	403	44.9
335.0	425	35	427	29.8	410	38.8
334.5	442	0.772	452	0.417	434	3.52
334.0	452	0.341	453	0.213	444	0.109
333.5	449	0.141	448	0.0918	441	0.0531
333.0	448	0.0857	440	0.046	438	0.0575
332.5	442	0.0524	436	0.0282	435	0.01168

Table A.4 continued from previous page

332.0	428	0.0257	427	0.0122	420	0.00945
331.5	405	0.0173	403	0.0072	397	0.0065
331.0	379	0.013	375		357	0.0067
330.5	324	0.0106	350		317	
330.0	212	0.00822	239		204	
329.5	50.6		49.4		73.9	
329.0	4.67		4.01		4.02	
328.5	23.8		25.6		37.9	
328.0	80.9		97.8		100.9	
327.5	132		143		140	
327.0	178		169		160	
326.5	209		194		193	
326.0	240		233		226	
325.5	257		255		243	
325.0	267		261		258	
324.5	271		269		261	
324.0	279		274		269	
323.5	285		281		274	
323.0	285		280		273	
322.5	287		284		276	
322.0	289		287		280	
321.5	288		289		279	
321.0	290		294		285	
320.5	300		298		292	
320.0	301		299		294	
319.5	304		302		296	
319.0	306		305		297	
318.5	310		307		298	
318.0	310		306		295	
317.5	306		303		293	
317.0	306		307		296	
316.5	309		303		297	
316.0	302		302		294	
315.5	300		296		290	
315.0	298		297		297	
314.5	293		292		299	
314.0	295		295		291	
313.5	290		289		293	
313.0	285		287		288	
312.5	279		280		288	
312.0	277		278		272	
311.5	269		268		276	
311.0	266		266		266	
310.5	264		267		272	

Table A.4 continued from previous page

310.0	264	261	262

Table A.5: Experiment 3: Horizontal Polarization Cont.

	Trial 4		Trial 5	
Angle	Reflected	Transmitted	Reflected	Transmitted
350.0	430	66.7	435	68.1
349.5	437	67.4	438	68
349.0	436	67.7	438	68
348.5	436	68	437	68.2
348.0	435	68	438	68.2
347.5	433	68.1	434	68.5
347.0	431	69.1	430	68.7
346.5	427	69.3	429	68.9
346.0	433	69.4	429	68.8
345.5	424	69	426	68.8
345.0	430	69	425	68.6
344.5	424	69.5	426	68.5
344.0	421	69.4	423	68.6
343.5	425	69.8	422	68.6
343.0	417	70.1	421	69
342.5	417	70	419	69.1
342.0	416	70.2	417	69.1
341.5	414	70.1	414	69.4
341.0	412	70	413	69.3
340.5	406	69.6	410	69.2
340.0	407	70.1	405	68.7
339.5	403	69.5	401	68.5
339.0	399	68.5	400	67.6
338.5	396	68	398	66.4
338.0	395	67	397	65.1
337.5	393	65.4	394	64.4
337.0	394	63.6	391	63
336.5	395	62	391	62.3
336.0	397	55.1	398	56
335.5	403	48.7	401	46.7
335.0	425	23.2	427	29.2
334.5	444	0.857	442	0.698
334.0	444	0.369	445	0.211
333.5	447	0.0152	445	0.0463
333.0	441	0.00631	440	0.00925
332.5	434	0.00701	439	0.00633
332.0	416	0.00524	422	0.00507
331.5	398		401	
331.0	370		363	
330.5	322		331	
330.0	187		179	

Table A.5 continued from previous page

329.5	30.2		45.6	
329.0	1.31		5.23	
328.5	54		29.2	
328.0	115		110	
327.5	160		158	
327.0	181		169	
326.5	206		199	
326.0	237		233	
325.5	263		247	
325.0	265		258	
324.5	270		269	
324.0	275		269	
323.5	280		280	
323.0	280		282	
322.5	281		282	
322.0	281		280	
321.5	283		285	
321.0	285		284	
320.5	291		292	
320.0	296		295	
319.5	296		296	
319.0	295		297	
318.5	296		295	
318.0	295		296	
317.5	292		295	
317.0	294		294	
316.5	293		292	
316.0	288		289	
315.5	283		285	
315.0	285		283	
314.5	280		279	
314.0	281		278	
313.5	275		275	
313.0	269		270	
312.5	266		265	
312.0	262		264	
311.5	254		255	
311.0	253		251	
310.5	251		249	
310.0	250		251	

Tables A.6 and A.7 are a continuation of the data from A.4 and A.5, but with vertical polarization.

Table A.6: Experiment 3: Vertical Polarization

	Trial 1		Trial 2		Trial 3	
Angle	Reflected	Transmitted	Reflected	Transmitted	Reflected	Transmitted
350.0	422	28.6	402	26.2	379	25.6
349.5	424	28.2	405	25.8	378	25.1
349.0	425	28	407	25.8	385	25.1
348.5	430	27.5	425	25.6	388	24.7
348.0	431	27.3	414	25.4	390	24.2
347.5	437	26.9	420	25	387	24.2
347.0	438	26.7	419	25	398	23.6
346.5	440	26.6	423	24.6	402	23.4
346.0	444	26.2	428	24	402	23.6
345.5	438	26.1	429	23.6	399	23
345.0	447	25.8	432	23.1	402	22.8
344.5	451	25.7	432	23.3	401	22.8
344.0	450	25.2	433	22.9	400	22
343.5	452	24.8	434	22.3	406	21.8
343.0	455	24.5	433	22.2	406	21.5
342.5	455	24.2	435	21.8	406	21.2
342.0	456	24	434	21.3	408	20.7
341.5	458	23.6	434	20.9	412	20.2
341.0	461	23.2	431	20.4	411	19.9
340.5	462	22.8	430	20.1	414	19.3
340.0	463	22.5	428	19.5	414	18.6
339.5	467	21.9	424	18.3	407	18.2
339.0	471	21.4	422	18.1	407	17.4
338.5	467	20.9	419	17.6	407	16.8
338.0	478	20.1	417	16.8	404	15.9
337.5	476	19.7	415	15.5	404	15
337.0	479	19.2	416	14.5	411	14.3
336.5	481	18.3	414	14.3	415	13.7
336.0	490	17.5	413	12.2	413	11.6
335.5	490	16.7	415	9.05	419	9.38
335.0	487	15.1	405	4.27	423	5.62
334.5	493	14.2	424	0.0299	423	0.0634
334.0	494	11.8	423	0.0271	427	0.0385
333.5	500	9.15	419	0.0174	430	0.0165
333.0	504	6.44	429	0.00899	427	0.00997
332.5	509	1.32	429	0.00819	424	0.00907
332.0	512	0.0995	426	0.00961	425	0.00825
331.5	512	0.0653	424	0.01162	419	0.00927
331.0	513	0.0492	427	0.0113	420	0.00916

Table A.6 continued from previous page

330.5	511	0.0521	409	0.0107	415	0.00912
330.0	512	0.0335	411	0.01083	395	0.00917
329.5	512	0.0372	399	0.0123	391	0.00909
329.0	503	0.0491	389	0.012	385	
328.5	493		408		398	
328.0	475		426		397	
327.5	458		428		413	
327.0	451		437		419	
326.5	454		436		405	
326.0	467		427		415	
325.5	476		425		419	
325.0	472		429		413	
324.5	475		433		414	
324.0	486		437		416	
323.5	490		438		423	
323.0	480		432		417	
322.5	491		427		406	
322.0	498		433		410	
321.5	496		434		412	
321.0	500		442		417	
320.5	499		451		424	
320.0	502		447		423	
319.5	499		455		425	
319.0	501		462		429	
318.5	504		457		426	
318.0	493		458		412	
317.5	507		453		422	
317.0	509		452		419	
316.5	504		458		413	
316.0	505		466		415	
315.5	504		471		412	
315.0	503		466		420	
314.5	504		471		415	
314.0	515		472		416	
313.5	497		459		413	
313.0	499		461		406	
312.5	492		462		382	
312.0	493		460		391	
311.5	479		450		389	
311.0	473		458		387	
310.5	483		458		385	
310.0	482		448		382	

Table A.7: Experiment 3: Vertical Polarization Cont.

	Trial 4		Trial 5	
Angle	Reflected	Transmitted	Reflected	Transmitted
350.0	388	26.2	362	25.4
349.5	394	26.1	372	25.5
349.0	392	26.1	374	25.3
348.5	396	25.9	373	25.2
348.0	395	25.5	383	25.2
347.5	400	24.8	391	25.8
347.0	400	24.6	391	24.6
346.5	393	24.3	393	24.5
346.0	403	23.9	397	24.2
345.5	404	23.6	396	23.8
345.0	406	23.3	401	23.4
344.5	404	22.8	400	22.8
344.0	404	22.5	403	22.8
343.5	406	22.2	406	22.3
343.0	405	21.8	405	22.1
342.5	405	21.5	405	21.6
342.0	407	21	404	21.5
341.5	406	20.5	405	20.9
341.0	407	19.9	405	20.3
340.5	408	19.8	403	19.8
340.0	406	19.1	403	19.3
339.5	405	18.5	404	18.7
339.0	404	17.9	399	18
338.5	404	17.3	400	17.3
338.0	401	16.3	397	16.3
337.5	401	14.9	397	15.1
337.0	402	14.2	400	14.2
336.5	404	13.4	393	13.5
336.0	402	12.1	399	12.2
335.5	406	9.32	402	9.31
335.0	410	6.54	401	6.19
334.5	411	0.0706	407	0.0688
334.0	411	0.0445	411	0.0431
333.5	415	0.0246	411	0.0227
333.0	414	0.0156	412	0.0174
332.5	411	0.0156	410	0.0162
332.0	409	0.0117	408	0.017
331.5	411	0.01136	402	0.0184
331.0	404	0.01096	392	0.0197
330.5	397	0.01018	384	0.0185
330.0	386	0.01069	376	0.0195

Table A.7 continued from previous page

329.5	370	0.0921	365	0.0194
329.0	376		365	0.0217
328.5	379		371	
328.0	387		377	
327.5	390		382	
327.0	396		387	
326.5	399		390	
326.0	398		390	
325.5	404		392	
325.0	402		391	
324.5	396		386	
324.0	398		386	
323.5	394		385	
323.0	395		381	
322.5	391		379	
322.0	394		378	
321.5	391		372	
321.0	390		374	
320.5	390		381	
320.0	393		382	
319.5	402		380	
319.0	406		386	
318.5	400		386	
318.0	402		387	
317.5	401		383	
317.0	402		385	
316.5	400		386	
316.0	394		379	
315.5	400		377	
315.0	394		374	
314.5	396		376	
314.0	393		374	
313.5	386		366	
313.0	383		363	
312.5	375		358	
312.0	370		356	
311.5	367		342	
311.0	363		345	
310.5	366		342	
310.0	361		338	

APPENDIX B: MATLAB CODE

This is the code used to calculate electron mobility in MATLAB.

```
format long
%For the calculation of electron mobility in the gold film
mass = 3.270*10^-25;% kg
conductivity = 4.10*10^7;% S/m
free_electrons = 5;% number of free electrons in valence shell
voltage = 1;% V
material_density = 19300;% kg/(m^3)
atomic_weight = 196.67;% mol/m^3
length = 0.01;% m
area = 0.02*0.00001;% cross-sectional area (m^2)
current = 1;% A
avog = 6.02*10^23;% Avogadro's number
elec_density = free_electrons*material_density/1000*atomic_weight*avog;
% number electrons per cumbic meter
elemcharge = 1.6021766*10^-19;% elementary charge e (in C)

drift_velocity = current / (elec_density * area * elemcharge) % m/s
%mobility = drift_velocity / field_density
field_density = elec_density*elemcharge*drift_velocity/conductivity % V/m
```